

A Small Scale Multi-Stage Hydro-Thermal Scheduling Problem

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Abstract

This document aims at presenting the correct solution to a small scale hydro-thermal scheduling problem presented in the literature and at the <http://stoprog.org/> website. The optimal solution suggested here is different from the ones in the literature and at the website. Hopefully, the presentation of the solution is transparent and correct. Please contact the author if you have comments or corrections to this document.

1 Example Description

Fig. 1 shows a test system with 4 hydropower modules, as described in [1]. The hydropower generation (variables p_1-p_4) has to be coordinated with one thermal unit (p_{th}) and a load curtailment opportunity, represented as an expensive dummy generator (p_d). Variables x_1-x_4 represent the storages in the corresponding reservoirs. The energy and cost units used in this example are not stated in the original example in [1]. Thus, one can refer to load, generation, inflow and hydro storage in any common energy unit and costs in any common monetary unit. A firm demand of 200 should be served in each time stage. Characteristics for the two systems are shown in Tables 1-2. The initial reservoir storages are all set to 10 % of the maximum storage capacities. Note that the example description here is identical to the one in [1].

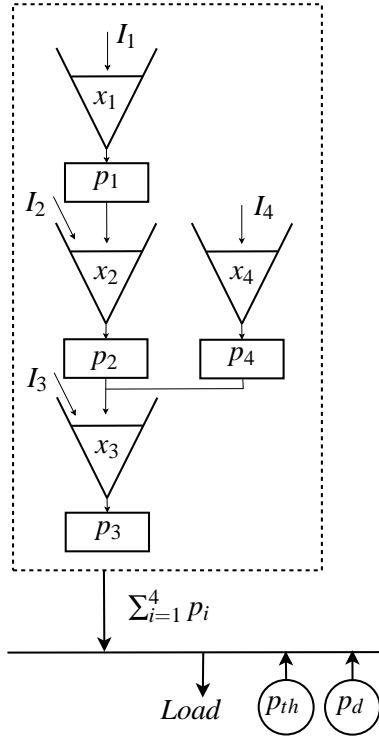


Figure 1: Hydro-thermal test system.

Table 1: Characteristics of the Hydropower Modules.

Hydro Plant	Maximum Storage	Maximum Outflow	Generation Efficiency
1	500	100	0.8
2	400	140	0.8
3	500	100	0.8
4	500	240	0.8

Table 2: Characteristics of the Thermal Generation System.

Thermal Plant	Maximum Capacity	Unit Cost
Plant 1	50	1
Curtailment	∞	10

Inflow (I) to the hydro reservoirs is treated as a stochastic variable. This example has three stages, where each node in the scenario tree has two succeeding branches with equal probability of occurrence. The scenario tree is illustrated in Fig. 2. The corresponding inflow scenarios are stated in Table 3.

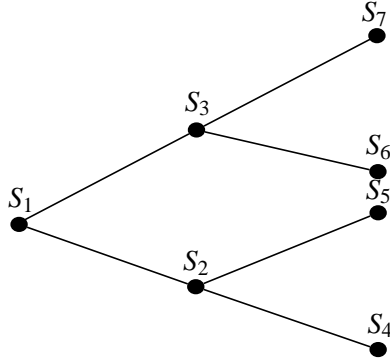


Figure 2: Enumerated scenario tree.

Table 3: Inflow records.

State	I_1	I_2	I_3	I_4
S1	10	12	10	5
S2	8	6	4	2
S3	7	5	3	9
S4	3	5	4	4
S5	2	9	3	5
S6	5	4	2	3
S7	10	11	8	9

2 Solution Proposal

The solution presented here is slightly different from the one in [1]. The problem was solved using two different approaches:

- a. Solving the deterministic equivalent as one LP problem
- b. Multi-stage Benders decomposition

In both cases an optimal objective function value of 318.7 was found. Note that the problem is degenerate, so comparing e.g. first-stage solutions has limited value. The LP problem (in the *.lp-format) is printed at the end of this document.

Below is the iteration log for the forward and backward iterations from the multi-stage Benders decomposition. The objective – excluding the future cost function variable – is shown for each state in the forward iteration. For the backward iterations, the aggregate cuts are shown. The left hand side coefficients refer to the state variables x_1 - x_4 .

```
1 --- Forward iteration no. 1 ---
2     Obj 1: 0.00
3     Obj 2: 558.00
4     Obj 3: 526.00
5     Obj 4: 630.00
6     Obj 5: 606.00
7     Obj 6: 614.00
8     Obj 7: 430.00
9 Upper bound: 1112.00
10 Lower bound: -10000000000.00
11 --- Backward iteration no. 1 ---
12 cut to state 3: 16.00 8.00 0.00 8.00 >= 522.00
13 cut to state 2: 16.00 8.00 0.00 8.00 >= 618.00
14 cut to state 1: 16.00 8.00 0.00 8.00 >= 1112.00
15
16 --- Forward iteration no. 2 ---
17     Obj 1: 310.00
18     Obj 2: 50.00
19     Obj 3: 50.00
20     Obj 4: 58.00
21     Obj 5: 46.00
22     Obj 6: 44.40
23     Obj 7: 6.80
24 Upper bound: 398.80
25 Lower bound: 46.00
26 --- Backward iteration no. 2 ---
27 cut to state 3: 2.40 1.60 0.80 1.60 >= 156.40
28 cut to state 2: 13.20 8.80 4.40 8.80 >= 736.20
29 cut to state 1: 7.80 5.20 2.60 5.20 >= 866.20
30
31 --- Forward iteration no. 3 ---
32     Obj 1: 50.00
33     Obj 2: 50.00
34     Obj 3: 36.71
35     Obj 4: 318.00
36     Obj 5: 270.00
37     Obj 6: 386.89
38     Obj 7: 46.09
39 Upper bound: 348.60
```

```
40 Lower bound: 223.30
41 --- Backward iteration no. 3 ---
42 cut to state 3: 13.20 8.80 4.40 8.80 >= 719.80
43 cut to state 2: 24.00 16.00 8.00 16.00 >= 1278.00
44 cut to state 1: 18.60 12.40 6.20 12.40 >= 1921.00
45
46 --- Forward iteration no. 4 ---
47     Obj 1: 50.00
48     Obj 2: 50.00
49     Obj 3: 50.00
50     Obj 4: 318.00
51     Obj 5: 270.00
52     Obj 6: 254.00
53     Obj 7: 32.80
54 Upper bound: 318.70
55 Lower bound: 318.70
```

```

1  \Problem name: Hydro-thermal scheduling problem (Pereira&Pinto 1985)
2
3  Minimize
4  OBJROW: x0 + 10 x1 + 0.5 x14 + 5 x15 + 0.5 x28 + 5 x29 + 0.25 x42 + 2.5 x43
5           + 0.25 x56 + 2.5 x57 + 0.25 x70 + 2.5 x71 + 0.25 x84 + 2.5 x85
6  Subject To
7  cons0:  - x2 - x3 - x4 = -60
8  cons1:  x3 + x4 - x5 - x6 - x7 = -52
9  cons2:  x6 + x7 - x8 - x9 - x10 + x12 + x13 = -60
10 cons3:  - x12 - x13 - x11 = -55
11 cons4:  x0 + x1 + 0.8 x3 + 0.8 x6 + 0.8 x9 + 0.8 x12 = 200
12 cons5:  x2 - x16 - x17 - x18 = -8
13 cons6:  x5 + x17 + x18 - x19 - x20 - x21 = -6
14 cons7:  x8 + x20 + x21 - x22 - x23 - x24 + x26 + x27 = -4
15 cons8:  x11 - x26 - x27 - x25 = -2
16 cons9:  x14 + x15 + 0.8 x17 + 0.8 x20 + 0.8 x23 + 0.8 x26 = 200
17 cons10: x2 - x30 - x31 - x32 = -7
18 cons11: x5 + x31 + x32 - x33 - x34 - x35 = -5
19 cons12: x8 + x34 + x35 - x36 - x37 - x38 + x40 + x41 = -3
20 cons13: x11 - x40 - x41 - x39 = -9
21 cons14: x28 + x29 + 0.8 x31 + 0.8 x34 + 0.8 x37 + 0.8 x40 = 200
22 cons15: x16 - x44 - x45 - x46 = -3
23 cons16: x19 + x45 + x46 - x47 - x48 - x49 = -5
24 cons17: x22 + x48 + x49 - x50 - x51 - x52 + x54 + x55 = -4
25 cons18: x25 - x54 - x55 - x53 = -4
26 cons19: x42 + x43 + 0.8 x45 + 0.8 x48 + 0.8 x51 + 0.8 x54 = 200
27 cons20: x16 - x58 - x59 - x60 = -2
28 cons21: x19 + x59 + x60 - x61 - x62 - x63 = -9
29 cons22: x22 + x62 + x63 - x64 - x65 - x66 + x68 + x69 = -3
30 cons23: x25 - x68 - x69 - x67 = -5
31 cons24: x56 + x57 + 0.8 x59 + 0.8 x62 + 0.8 x65 + 0.8 x68 = 200
32 cons25: x30 - x72 - x73 - x74 = -5
33 cons26: x33 + x73 + x74 - x75 - x76 - x77 = -4
34 cons27: x36 + x76 + x77 - x78 - x79 - x80 + x82 + x83 = -2
35 cons28: x39 - x82 - x83 - x81 = -3
36 cons29: x70 + x71 + 0.8 x73 + 0.8 x76 + 0.8 x79 + 0.8 x82 = 200
37 cons30: x30 - x86 - x87 - x88 = -10
38 cons31: x33 + x87 + x88 - x89 - x90 - x91 = -11
39 cons32: x36 + x90 + x91 - x92 - x93 - x94 + x96 + x97 = -8

```

```
40  cons33: x39 - x96 - x97 - x95 = -9
41  cons34: x84 + x85 + 0.8 x87 + 0.8 x90 + 0.8 x93 + 0.8 x96 = 200
42  Bounds
43  0 <= x0 <= 50
44  0 <= x1 <= 10000000000
45  0 <= x14 <= 50
46  0 <= x15 <= 10000000000
47  0 <= x28 <= 50
48  0 <= x29 <= 10000000000
49  0 <= x42 <= 50
50  0 <= x43 <= 10000000000
51  0 <= x56 <= 50
52  0 <= x57 <= 10000000000
53  0 <= x70 <= 50
54  0 <= x71 <= 10000000000
55  0 <= x84 <= 50
56  0 <= x85 <= 10000000000
57  0 <= x2 <= 500
58  0 <= x3 <= 100
59  0 <= x4 <= 10000000000
60  0 <= x5 <= 400
61  0 <= x6 <= 140
62  0 <= x7 <= 10000000000
63  0 <= x8 <= 500
64  0 <= x9 <= 100
65  0 <= x10 <= 10000000000
66  0 <= x12 <= 240
67  0 <= x13 <= 10000000000
68  0 <= x11 <= 500
69  0 <= x16 <= 500
70  0 <= x17 <= 100
71  0 <= x18 <= 10000000000
72  0 <= x19 <= 400
73  0 <= x20 <= 140
74  0 <= x21 <= 10000000000
75  0 <= x22 <= 500
76  0 <= x23 <= 100
77  0 <= x24 <= 10000000000
78  0 <= x26 <= 240
79  0 <= x27 <= 10000000000
```

```
80 0 <= x25 <= 500
81 0 <= x30 <= 500
82 0 <= x31 <= 100
83 0 <= x32 <= 10000000000
84 0 <= x33 <= 400
85 0 <= x34 <= 140
86 0 <= x35 <= 10000000000
87 0 <= x36 <= 500
88 0 <= x37 <= 100
89 0 <= x38 <= 10000000000
90 0 <= x40 <= 240
91 0 <= x41 <= 10000000000
92 0 <= x39 <= 500
93 0 <= x44 <= 500
94 0 <= x45 <= 100
95 0 <= x46 <= 10000000000
96 0 <= x47 <= 400
97 0 <= x48 <= 140
98 0 <= x49 <= 10000000000
99 0 <= x50 <= 500
100 0 <= x51 <= 100
101 0 <= x52 <= 10000000000
102 0 <= x54 <= 240
103 0 <= x55 <= 10000000000
104 0 <= x53 <= 500
105 0 <= x58 <= 500
106 0 <= x59 <= 100
107 0 <= x60 <= 10000000000
108 0 <= x61 <= 400
109 0 <= x62 <= 140
110 0 <= x63 <= 10000000000
111 0 <= x64 <= 500
112 0 <= x65 <= 100
113 0 <= x66 <= 10000000000
114 0 <= x68 <= 240
115 0 <= x69 <= 10000000000
116 0 <= x67 <= 500
117 0 <= x72 <= 500
118 0 <= x73 <= 100
119 0 <= x74 <= 10000000000
```

```
120 0 <= x75 <= 400
121 0 <= x76 <= 140
122 0 <= x77 <= 10000000000
123 0 <= x78 <= 500
124 0 <= x79 <= 100
125 0 <= x80 <= 10000000000
126 0 <= x82 <= 240
127 0 <= x83 <= 10000000000
128 0 <= x81 <= 500
129 0 <= x86 <= 500
130 0 <= x87 <= 100
131 0 <= x88 <= 10000000000
132 0 <= x89 <= 400
133 0 <= x90 <= 140
134 0 <= x91 <= 10000000000
135 0 <= x92 <= 500
136 0 <= x93 <= 100
137 0 <= x94 <= 10000000000
138 0 <= x96 <= 240
139 0 <= x97 <= 10000000000
140 0 <= x95 <= 500
141 End
142
```

References

- [1] M. V. F. Pereira and L. M. V. G. Pinto. Stochastic optimization of a multireservoir hydroelectric system: A decomposition approach. *Water Resources Research*, 21(6):779–792, 1985.